Quantifying Geotechnical Probability of Failure—a Simpler Approach

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ABSTRACT

Geotechnical engineers test and explore less than one millionth of a site's soil or rock, and based on that information predict how the remaining 999,999 parts will behave under new loads that the project creates. Prediction errors occur from the variability of the site's soil/rock and the accuracy of the chosen design method. The author presents a design method to quantify the probability of successfully predicting the desired outcome that the owner can understand and choose. Engineers must design to prevent loss of life with very low probabilities of failure, but may minimize financial probability of failure of undesired performance with a higher probability of failure to match the owner's desires. Following Dr. Briaud's writing style for technical papers, the engineer can quantify probability of failure accurately without cumbersome calculations.

INTRODUCTION

Usually geotechnical engineers qualify probability of failure from their experience-based "engineering judgment". Their experience comes primarily from observing constructed projects that did not fail. However, true experience comes from observing failures, whether unplanned or planned, such as load tests. Most geotechnical engineers only have "conservative" and perhaps "overly conservative" judgment because they have not observed enough failures.

Fortunately, the geotechnical engineer can quantify probability of failure using probability methods. While many technical papers present complicated mathematical formulas to compute probability of failure, this paper presents simple charts with linear relationships to compute probability of failure. Project owners can understand these charts enabling the engineer to effectively communicate probability of failure with the owner.

Often engineers use the same field and laboratory tests for every project. Instead they should choose the tests that accurately measure the soil or properties and design methods that accurately predict outcomes. For example, the cone penetrometer test (CPT), a miniature pile, provides high quality and quantity of data for predicting the vertical capacity of deep foundations. The dilatometer test (DMT) and pressuremeter test (PMT), which are static deformation tests, provide high quality data for settlement predictions of shallow foundations. Additionally, both DMT and PMT expand laterally against the soil, and accurately predict the lateral capacity of deep foundations. From case study literature, the engineer can obtain the coefficient of variation for the predicted/observed ratio to determine the accuracy of the test and design method.

BETA PROBABILITY DISTRIBUTION FOR PROBABILITY OF FAILURE MODELLING

Numerous probability distribution functions have been published to model civil engineering problems. While many engineers use the normal and log normal distributions, both distributions have unrealistic end limits. The normal distribution ranges from negative infinity to positive infinity and the log normal distribution ranges from slightly more than zero to positive infinity. With the beta distribution, the engineer chooses its end limits.

The area underneath a probability distribution curve must always equal 1.0 because the likelihood of the event occurring is 100% whether a positive or negative outcome. The probability of success equals 1.0 minus the probability of failure. The engineer integrates the area under one of the tails of the distribution function to quantify the probability of failure or undesired consequence. In the case of a factor of safety solution, the area under the left tail less than 1.0 quantifies the probability of failure. For the solution to a settlement problem, the area under the right tail that exceeds the maximum desired settlement quantifies the probability of failure or undesired consequence.

Realistic choices of the end limits critically define the probability of failure. The author chose end limits as the average value plus or minus either 3 or 5 times its standard deviation with the additional restriction that the minimum value could not be less than zero. The equation for a beta probability distribution function requires four input parameters: the minimum end limit, the maximum end limit, the average value and its standard deviation. The area under this curve must equal 1.0, and the engineer can check his/her equation by numerically integrating using the trapezoidal method.

For the factor of safety problem, if one desires a probability of success equal to 95% and an average factor of safety of 1.2 with its end limits equal to plus or minus 3 standard deviations, then he/she can solve for the unique standard deviation. One solves this equation easier by numerically integrating under the left tail where the factor of safety is less than 1.0 and the probability of failure equals 5% or its area equals 0.05.

The author selected different values of factors of safety and solved for their standard deviations so that the probability of failure equaled 0.05. When plotting this data set with the average value on the y-axis and its standard deviation on the x-axis, the probability of success for 95% followed a linear relationship. For different values of probability of success (90%, 99%, 99.9%, 99.99% and 99.999%), their solutions also resulted in linear relationships. Similarly for end limits equal to 5 times their standard deviation plus or minus the average value, their solutions were linear. Solutions for settlement examined the area under the right tail that exceeded the maximum tolerable amount as the failure area and those solutions also were linear.

The minimum end limit could not be less than zero and zero was used when the average value minus either 3 or 5 times the standard deviation would have equaled a negative number. In these cases, the beta probability distribution became skewed right instead of symmetric and the solution became non-linear.

EVALUATING STANDARD DEVIATION

Geotechnical engineers predict an outcome based on limited data. Ideally, the uncertainty in their prediction should arise from only the natural variability of the soil or rock properties. However, often the test for the properties and the design method have uncertainty. By comparing predicted values with actual measured values from case study reports, the engineer can obtain the coefficient of variation (standard deviation divided by the average value) for the chosen design method. Additional uncertainty can occur when an owner selects the contractor by low bid instead of choosing by pre-qualifications, choses another firm to inspect or monitor the construction or limits the number of property tests. The engineer should use his/her engineering judgement to quantify this source of uncertainty. If these three sources of uncertainty (natural variability, design method and other) are considered to be independent of each other, then the overall standard deviation equals the square root of the sum of the individual standard deviations squared. If these sources have some dependence with each other, then the overall standard deviation is less than that.

FACTOR OF SAFETY

Figure 1 shows the beta probability distribution curves for probabilities of success equal to 95% with end limits equal to the average factor of safety plus or minus 3 standard deviations. The area under each curve where the factor of safety was less than 1.0 equaled 0.05. As uncertainty decreases, the curves become narrow and steep; but when uncertainty increases, the curves become wide and flat. Figure 2 presents a design chart showing the linear equations for different values of probability of success when the end limits equal the average value of factor of safety either plus or minus 3 standard deviations. Figure 3 presents the design chart with end limits equal to the average factor of safety plus or minus 5 standard deviations.

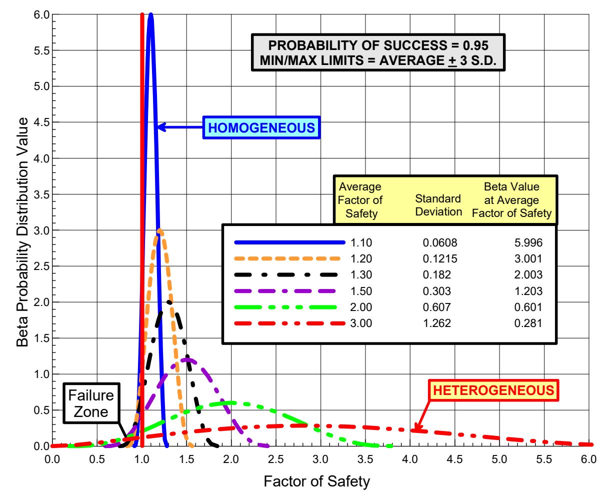

Figure 1: Beta Probability Distribution Functions for Probability of Success = 95%

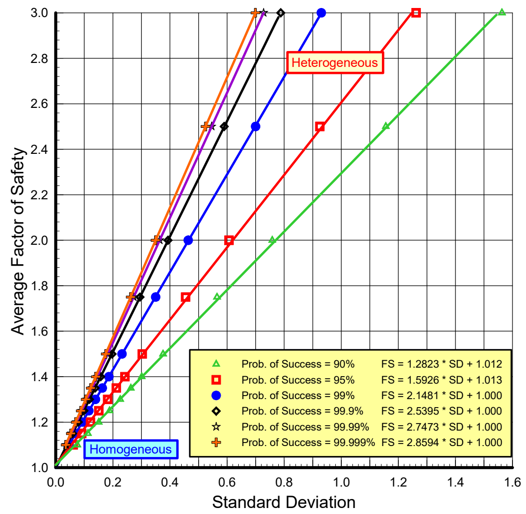

Figure 2: Design Chart for Factor of Safety with end limits = average $\pm$ 3 standard deviations

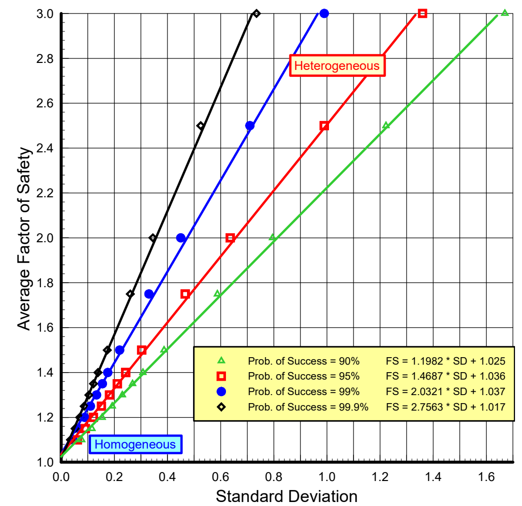

Figure 3: Design Chart for Factor of Safety with end limits = average $\pm$ 5 standard deviations

Knowing the average value of the factor of safety without knowing its standard deviation means nothing when evaluating probability of failure. The point estimate method presented by Christian (1997) offers a reasonable approach for determining the standard deviation. For each variable such as shear strength of a stratum or groundwater level, one uses its average value either plus or minus its standard deviation and solves for the factor of safety. This approach provides a data set with 2^n points, with n being the number of variables. From this data set one computes the average factor of safety and standard deviation and plots that point on Figure 2 or 3 to determine the probability of success. When one has difficulty determining a property's standard deviation, Duncan (2000) recommends choosing the maximum and minimum possible values and dividing their difference by 6. He suggests that end limits equal to 3 standard deviations away from the average value represent realistic values for geotechnical engineering design.

Slope Stability Example: This example describes a hypothetical slope stability design using electric cone penetration tests, performed during a phase one subsurface investigation that delineated three geologic strata at the site. The phase two investigation included five borehole shear tests performed in each stratum to estimate the average drained strength parameters and their standard deviations. The borehole shear test measures the drained shear strength of the soil and compares well to laboratory strength tests (Handy, 1986).

The point estimate method (Christian, 1997) assigned a value of either the average plus one standard deviation or the average minus one standard deviation to each variable. Using the shear strength of each of the three strata and the groundwater level as the parametric variables, multiple runs with a Janbu stability analysis program provided a total of 16 permutations (2^n, where n = the number of variables = 4) of factor of safety values. For these permutations, the average factor of safety equaled 1.25 with a standard deviation of 0.15. The design chart in Figure 2 indicates an acceptable 95% probability of success.

Ground Improvement Evaluation: In a case study (Miller and Roycroft, 2004) compaction grouting was performed to densify a loose sand to prevent liquefaction of the site. The test program used both 1.2 and 1.5 m spacing between the grouting locations. Afterwards, numerous cone penetration test soundings (CPT) were performed, and the factors of safety against liquefaction were computed. For the 1.5 m spacing the average factor of safety was 1.51, and for the 1.2 m spacing the average factor of safety was 1.65. Based on their engineering judgment, the authors recommended using a minimum acceptable factor of safety of 1.2 and concluded that the 1.5 m spacing was acceptable.

However, from the large amount of data that the authors had collected, the standard deviation of the factor of safety was 0.47 for the 1.5 m spacing and 0.41 for the 1.2 m spacing. These standard deviations values are rather high showing the heterogeneity or high uncertainty of the sands for liquefaction resistance. When plotted on Figure 2, the probability of failure analysis for 1.5 m spacing yields a probability of success less than 90% (85% numerically computed) and the analysis for 1.2 m spacing yields a probability of success equal to 95%.

LOAD AND CAPACITY PROBABILITY DISTRIBUTION FUNCTIONS

If the load also has uncertainty, one can create a probability distribution function for it. In this case the probability of failure equals the curved triangular shape where the load curve exceeds the capacity curve (Harr, 1977). The author considered load curves with standard deviations equal to 0.1 and 0.2 times the average load. Assuming an average load equal to 1.0, the analyses became unitless. Figures 4 and 5 show the beta distribution curves for a probability of success equal to 95% and maximum and minimum end limits equal to the average value plus or minus 3 standard deviations for load with standard deviations of 0.1 and 0.2, respectively. Again, similar to the factor of safety solution presented above, with the factor of safety or capacity/load ratio as the y-axis and the capacity standard deviation as the x-axis, the data plotted as linear relations for different probabilities of success. Figures 6 and 7 present design charts for these lines for load standard deviations of 0.1 and 0.2, respectively.

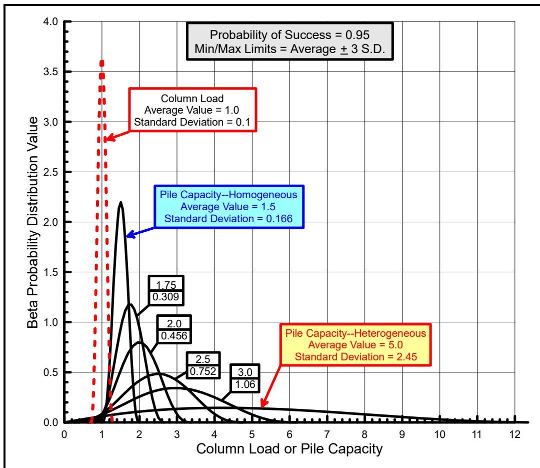

Figure 4: Beta Distribution Curves with Load Curve with Standard Deviation of 0.1

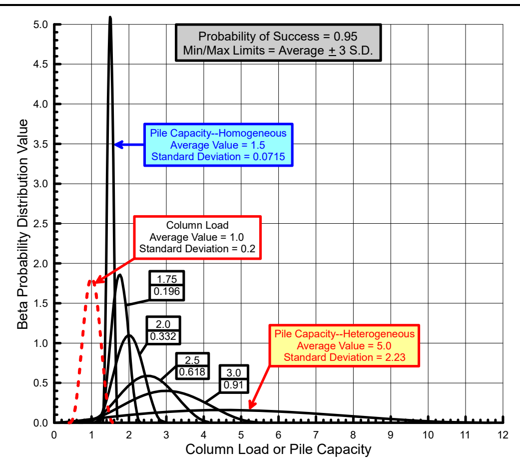

Figure 5: Beta Distribution Curves with Load Curve with Standard Deviation of 0.2

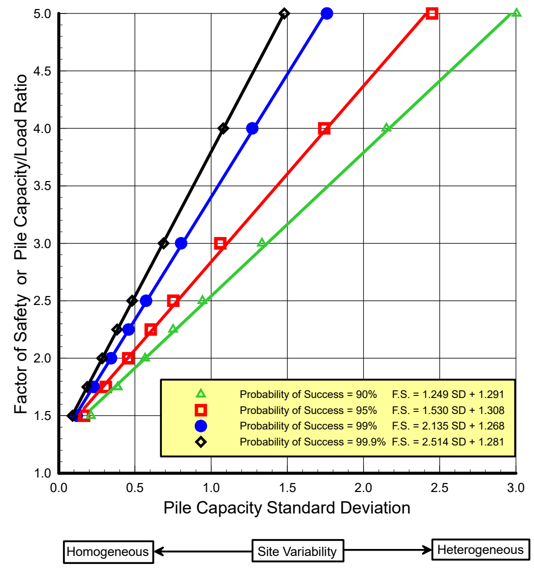

Figure 6: Design Chart for Load Standard Deviations of 0.1

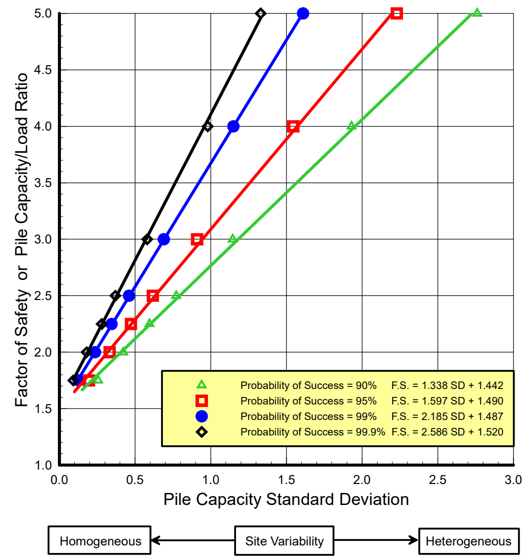

Figure 7: Design Chart for Load Standard Deviations of 0.2

Pile Capacity Example: This example considers twenty (20) cone penetrometer test soundings performed for the hypothetical design of a laboratory foundation supported by steel pipe piles. Column loads will require support of 200 kN per pile, with a standard deviation of 20 kN. For each sounding the LCPC pile capacity prediction method provided a pile designed to carry a load of 350 kN (nominal safety factor = 1.75). The design tip elevations for the different column loads in the foundation plan did not vary greatly, resulting in a standard deviation of only 35 kN due to the natural soil variability. Based on a database case study, Robertson, et al. (1988) indicate a coefficient of variation of 0.15 for the LCPC predicted capacity of driven steel pipe piles. Using this value, the standard deviation due to the LCPC method is 0.15 * 350 kN = 52.5 kN. The overall standard deviation equals the square root of the sum of the two individual standard deviations squared, or 63.1 kN. The columns were designed to exert a load of 200 kN per pile. Dividing by the 200 kN nominal applied load results in a unitless predicted pile capacity of 1.75, with a standard deviation of 0.32 and load standard deviation of 0.1. Because the building will contain sensitive laboratory equipment, the owner chose a 99% probability of success. However, Figure 6 indicates a probability of success of only 93% for the above parameters.

By increasing the pile diameter so that each pile will have a capacity of 400 kN, the natural standard deviation of 35 kN and the LCPC method standard deviation of 0.15 * 400 kN or 60 kN result in an overall standard deviation of 69.5 kN. Using the unitless values of 2.0 for the factor of safety and 0.35 for pile standard deviation, Figure 6 indicates an acceptable probability of success of 99%.

SETTLEMENT

Engineers often consider total settlements exceeding 1.0 inch (25 mm) as unsatisfactory. Again, one can use the beta probability distribution to assess the probability of failure. In this case, the probability of failure or undesired consequence equals the area under the right tail of the curve that exceeds the threshold value. Figure 8 shows the beta probability distribution curve for total settlement with a threshold value of 25 mm.

Alternatively, the engineer could examine angular distortion exceeding a threshold value depending on the type of construction and use as undesirable (Table 1). The geotechnical engineer, the owner and structural engineer, working closely together, should choose the appropriate probability of failure levels for angular distortion.

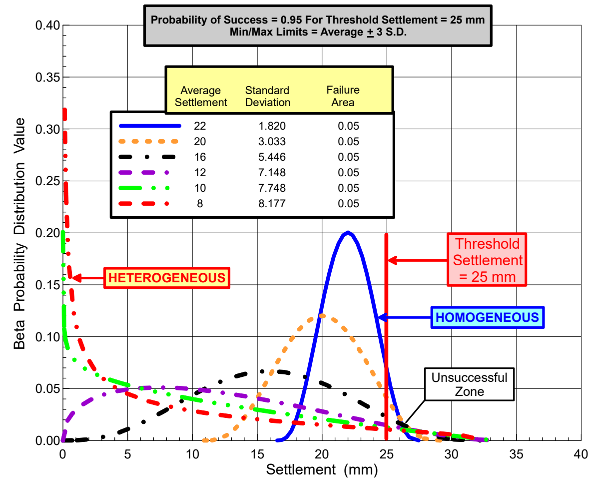

Figure 8: Beta Probability Distribution Curves for Settlement

Table 1: Allowable Angular Distortion

Situation	Allowable Angular Distortion
Machinery sensitive to settlement	1/750
No cracking in buildings; tilt of bridge abutments; tall slender structures such a stacks, silos, and water tanks on a rigid mat; steel or reinforced concrete frame with brick block, plaster or stucco finish and length to height ratio greater than 5	1/500
Cracking in panel walls; problems with overhead cranes	1/300
Structural damage in buildings; flexible brick walls with length to height ratio greater than 4	1/150

Using minimum and maximum end limits equal to the average settlement or angular distortion either minus or plus 3 standard deviations and plotting the average settlement/angular distortion on the y-axis and standard deviation on the x-axis, the data have a linear relationship as long as the curve is symmetric. When the solution forced the minimum value three standard deviations less than the average value to equal a negative value, the author used zero for the minimum end limit. In this case, the curve was skewed right and became reverse “J” shaped as the minimum limit and average settlement decreased. Figure 9 presents a design chart for total settlement analyses with a threshold value of 25 mm. Figures 10-13 present design charts for angular distortion with threshold values of 1/150, 1/300, 1/500 and 1/750, respectively.

Example Angular Distortion Problem: The owner plans to construct a four story reinforced concrete office building with drywall interior walls. He desires probabilities of success of 99.99% against structural damage and 95% against drywall cracking. The geotechnical engineer performs dilatometer soundings at most of the column locations and determines the average value of angular distortion of 0.0022 and a standard deviation of 0.0002. The coefficient of variation for the dilatometer prediction method = 0.18 (case study database); for the loads = 0.20 (provided by the structural engineer); for the contractor/inspector = 0.10 (selected by qualifications). The standard deviation equals the coefficient of variation times the average value of angular distortion. Therefore, the standard deviation for the DMT

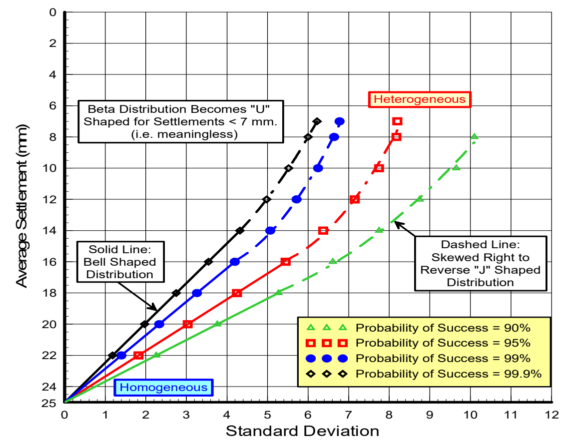

Figure 9: Design Chart for Probability of Success for Settlement

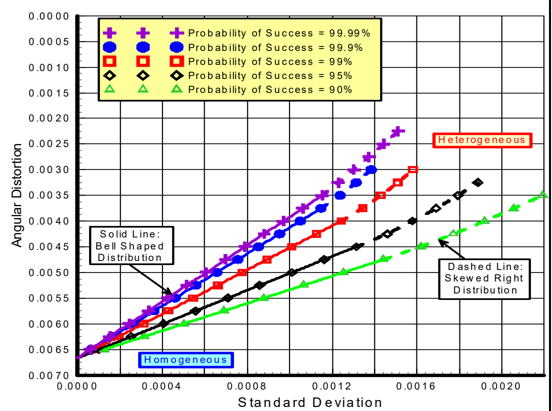

Figure 10: Design Chart for Probability of Success for 1/150 Angular Distortion

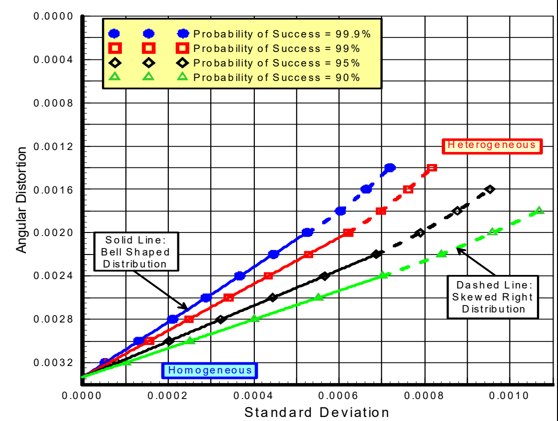

Figure 11: Design Chart for Probability of Success for 1/300 Angular Distortion

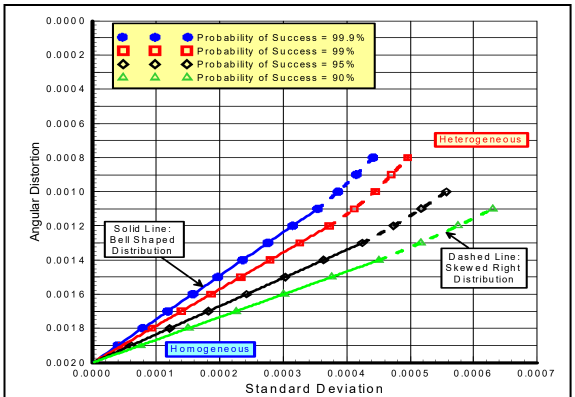

Figure 12: Design Chart for Probability of Success for 1/500 Angular Distortion

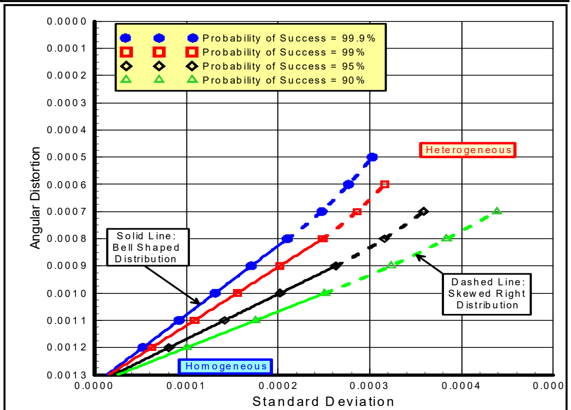

Figure 13: Design Chart for Probability of Success for 1/750 Angular Distortion

method = 0.0040, for the loads = 0.0044, and for the contractor/inspector = 0.0022. The overall standard deviation was 0.0063. By plotting the average and overall standard deviation values on Fig. 12 (angular distortion = 1/500), we estimate a 96% probability of success. These values plot significantly to the left of the 99.99% line on Fig. 10 (angular distortion = 1/150). Therefore, the design satisfies the owners' needs.

OWNER INVOLVEMENT

The total cost of a project equals 1) the construction cost, 2) the probability of failure times the cost to fix the failure, and 3) maintenance costs. The geotechnical engineer should work closely with the owner and structural engineer to minimize the costs of items 1 and 2. Successful owners understand and accept the project's probability of failure and will choose the most appropriate value for probability of success. The owner can choose a lower probability of success (90% or perhaps lower) if he determines his savings from the less conservative approach will be greater than the remedial fixes that may occur in hopefully isolated areas.

Their choice depends on many factors such as the intended use and sensitivity of the facility, foundation redundancy, costs to repair, installation of performance monitoring instruments, and quality of the contractor. A structure with equipment sensitive to differential settlement should use a probability of success of 99 or 99.9%, whereas a warehouse that can tolerate more differential settlement and still function adequately can tolerate a lower 90% probability of success. Pile supported structures that have some redundancy can also use lower probabilities of success, 90 or 95%. If one pile does not have its full desired capacity, a nearby pile may have additional capacity and provide the needed extra load capacity. Often pile groups need a whole number plus a fraction of a pile to carry the design load, but the additional pile is installed resulting in supplemental capacity (e.g. compute 8.2 piles, install 9 piles). A slope's location may help decide its appropriate probability of success. Highway departments may construct slopes with lower level of success, choosing to save money by repairing the occasional failed slope rather than buying more right-of-way to build flatter slopes. However, on a heavily traveled road, a higher probability of success reduces the probability of failure of a costly traffic delay. Instruments can be installed to monitor the performance of construction. Unsatisfactory areas can be detected and stabilized. The quality of the contractor and of the engineering inspection may also influence the design probability of success. High quality contractors and inspectors will recognize and correct for unanticipated subsurface conditions, providing a better product less susceptible to damage. The engineer should work with the owner to initially pre-qualify contractors and later help the owner select a contractor that has submitted a responsive bid.

The engineer must educate the owner on the design process and explain why certain tests will be conducted and how that knowledge will be used for improved design. By being involved with the owner, the engineer will develop and improve their business relationship. The owner will not consider the engineer as a commodity service (hiring and selecting the engineer based on a fee) but rather as a valuable contributor to his project. If the owner does not want to assume his/her probability of failures and the engineer loses the project, the engineer has only lost a bad client.

CONCLUSIONS

1. Good design requires that the owner accept and understand the probability of failure.
2. Effective probability of failure analysis requires the engineer to limit variability, as best possible, to that inherent in the geologic deposit.
3. A thorough and accurate site investigation helps to minimize design variability and improves design efficiency.
4. Soil tests that directly measure design parameters reduce variability better than empirical correlations with indirect measurements.
5. For a given probability of success, using the Beta probability distribution within common engineering limits provides a linear relationship between the average value of the design parameter and its standard deviation.

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