Risk Quantification for Design

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ABSTRACT: Geotechnical design requires determining the soil's properties and the uncertainty of their values. In traditional design, uncertainty is qualitatively determined and is commonly referred to as engineering judgment. However, a better assessment of uncertainty can be made if it is quantitatively determined, which can be done through a probability assessment. Geotechnical engineers often view the mathematics associated with a probability assessment as too complicated to perform. Extensive probability analyses were performed as the background for this paper. When the results were plotted, linear relationships for different probabilities of success were discovered. Now engineers can use the presented summary design charts to easily quantitatively determine uncertainty. Equally important, they can explain uncertainty to their client and the owner and then design for the desired risk level that the owner chooses.
Site investigation methods that improve the accuracy of design parameters will reduce risk, and the design will then focus on the site's true soil variability without parasitic test variability. Four examples illustrate geotechnical risk analysis using the Beta distribution and emphasize the importance of minimizing testing variability.

1 INTRODUCTION

Geotechnical engineers traditionally use their "engineering judgment" to qualitatively evaluate uncertainty in their design. Engineering judgment is gained by observing failures, either unplanned events or planned events such as load tests. Failures can either be catastrophic or unsatisfactory performance. Unsatisfactory performance is when a structure moves more than desired. Often engineers have not observed enough failures, and their judgment is not good enough. It is difficult for clients or owners to understand engineering judgment and often view geotechnical engineering as a voodoo science or commodity service instead of a professional service. Furthermore, if geotechnical recommendations are not what they expect, then they may question the design and the geotechnical engineer's judgment.

Having a project built and perform successfully only misleads engineers into believing that they are developing good engineering judgment. However, they have only learned that the design was conservative. It could also be overly conservative and unnecessarily costly. Thus engineering judgment cannot be improved without observing failures.

A better approach is to quantify uncertainty with probability methods. Engineering judgment can also be included in this method when evaluating the standard deviations of the design parameters. Probability distribution functions can define the relationship between variability and design performance risk. In the examples presented below, probability analyses using the Beta distribution indicate a nearly linear relationship between the nominal design factor of safety and the standard deviation (variability) for different probabilities of success. Note to improve client relations, the engineer should use the probability of "success" rather than its complement the probability of "failure".

After choosing a design factor of safety and computing its standard deviation, the engineer can use the figures below (or develop additional figures) to determine the probability of success. Adjustments to the design can then achieve the desired value. However, although greater safety minimizes the risk of sudden failure, excessive settlement, lateral movement, etc., it also results in higher construction costs.

An economical site investigation leading to reasonable design safety usually requires two phases, the first using rapid, less expensive insitu tests to identify and map critical areas, and the second providing more detailed tests of selected soil strata and design analysis. The latter should replace any preliminary analyses and quantify risk. Site variability, soil test accuracy, and the accuracy of the design method all affect the reliability of the final design.

2 OWNER INVOLVEMENT

The engineer must always design to avoid loss of life. However, all other design decisions are purely economical and the owner should decide the appropriate level of success. After all, it is the owner's money and soil. The owner is well versed with risk, for every financial decision he makes involves risk.

Because engineers typically do not discuss the possibility of failure with the owner, they assume liability that should remain with the owner. To cover the engineer's perceived liability, his designs are often overly conservative and costly and serve neither the owner nor the engineer. If the owner thinks the foundation costs will be too high, he will hire a second engineer and the first engineer may lose the project (perhaps, rightfully so). The owner and the engineer should mutually decide on the acceptable probability of success for the design. The owner's understanding and acceptance of the inherent risk help determine the feasibility of the project.

Our analyses below show results for success probabilities of 90%, 95%, 99%, and 99.9%. Choosing the most appropriate value depends on many factors such as the intended use and sensitivity of the facility, foundation redundancy, costs to repair, installation of performance monitoring instruments, and quality of the contractor. A structure with equipment sensitive to differential settlement should use a probability of success of 99 or 99.9%, whereas a warehouse that can tolerate more differential settlement and still function adequately can tolerate a lower 90% probability of success.

Pile supported structures that have some redundancy can also use lower probabilities of success, 90 or 95%. If one pile does not have its full desired capacity, a nearby pile may have additional capacity and provide the needed extra load capacity. Often pile groups need a whole number plus a fraction of a pile to carry the design load, but the additional pile is installed resulting in supplemental capacity (e.g. compute 8.2 piles, install 9 piles).

A slope's location may help decide its appropriate probability of success. Highway departments may construct slopes with lower level of success, choosing to save money by repairing the occasional failed slope rather than buying more right-of-way to build flatter slopes. However, on a heavily traveled road, a higher probability of success reduces the risk of a costly traffic delay.

Instruments can be installed to monitor the performance of construction. Unsatisfactory areas can be detected and stabilized. The owner can choose a lower probability of success (90% or perhaps lower) if he determines his savings from the less conservative approach will be greater than the remedial fixes that may occur in hopefully isolated areas.

The quality of the contractor and the quality of the engineering inspection may also influence the design probability of success. High quality contractors and inspectors will recognize and correct for unanticipated subsurface conditions, providing a better product less susceptible to damage. The engineer should work with the owner to initially pre-qualify contractors and later help the owner select a contractor that has submitted a responsive bid.

The engineer must educate the owner on the design process and explain why certain tests will be conducted and how that knowledge will be used for improved design. By being involved with the owner, the engineer will develop and improve their business relationship. The owner will not consider the engineer as a commodity service (hiring and selecting the engineer based on fee) but rather as a valuable contributor to his project. If the owner does not want to assume his risks and the engineer loses the project, the engineer has only lost a bad client.

3 EVALUATING STANDARD DEVIATION

The sources of standard deviation that affect risk assessment include the natural variability of the soil or rock, man-created variability added during the design process and intangible variability created by the owner. The engineer should carefully define the properties and boundaries for the different geologic formations at the site. A large number of measurements should be made to minimize the uncertainty for the standard deviation of a soil or rock property.

Man-created variability is how well the test/design predicts what will occur. The designer should attempt to minimize the man-created variability by using a sufficient quantity of tests that accurately predict the design parameters and by choosing design methods that accurately predict performance. This value can be quantified from case study databases.

The owner can create intangible variability by selecting a low bid contractor that is not pre-qualified or responsive, another firm to perform the inspection or not letting the engineer perform enough tests. This value is chosen based on engineering judgment.

If man-created and intangible sources of variability can be minimized, the engineer can focus on the geologic variability of the site, defining areas of poor or favorable geologic conditions and possibly designing those areas of the site separately. If the sources of variability are considered to be independent of each other, then the overall standard deviation equals the square root of the sum of the individual standard deviations squared. If the sources of variability are considered to be somewhat dependent on each other, a lower value of overall standard deviation may be used based on engineering judgment. As design uncertainty is reduced, the overall standard deviation decreases and a more efficient design occurs.

Duncan (2000) suggests that engineering judgment may be used to determine standard deviation (3-sigma rule). The engineer must decide what are the minimum and maximum possible values for a design parameter. An estimate for standard deviation is their difference divided by 6.

4 WHY USE THE BETA PROBABILITY DISTRIBUTION FUNCTION?

The area under any probability distribution function must equal 1.0. There is 100% chance or 1.0 that the event will occur. For example, if one flips a coin, it will land on either a heads or a tails. We don't know which one, but one will occur.

In engineering design probability distribution functions tend to be "bell-shaped." The probability of failure is the area under one of the tails of the probability distribution function. The probability of success will be the remaining area or 1.0 minus the probability of failure.

Normal or log-normal population distributions have often been used for engineering design, but they may not be appropriate for geotechnical design. For a normal distribution the minimum and maximum limits are negative and positive infinity, respectively. The log-normal distribution uses limits of zero and positive infinity. In either case, these limits are unrealistic and often impractical. With the more versatile Beta distribution (of which the normal distribution is a specific subset) the engineer chooses the minimum and maximum limits. In our analyses we evaluated both 3 and 5 standard deviations from the mean as the minimum and maximum limits. Because there was little difference in those results, we concur with the recommendation by Harr (1977) and Duncan (2000) to use minimum and maximum limits of 3 standard deviations away from the mean.

Steep and narrow Beta probability curves (with low standard deviations) describe homogeneous soil conditions, and flatter curves indicate imprecise or heterogeneous conditions. The following examples illustrate the evaluation of project risk for slope stability, ground improvement, vertical pile capacity, and settlement.

5 SLOPE STABILITY ANALYSIS/OTHER FACTOR OF SAFETY DESIGN APPROACHES

Many local and national codes specify a minimum factor of safety for earthen slopes. These specifications are often overly generalized and somewhat arbitrary because they seldom consider the homogeneity of the subsurface conditions or the consequences of failure. The owner does not want to buy excess land to have overly conservative slopes nor does he want to have an expensive repair later. The Beta probability distribution allows engineers to consider the above conditions in their analyses. The area under the probability curve with a factor of safety less than 1.0 defines the probability of failure. Homogeneous subsurface conditions, with low uncertainty (standard deviation), will result in a sharply peaked and narrow Beta distribution curve at a given probability of success, with an average factor of safety slightly more than 1.0. Conversely, a heterogeneous subsurface, with high uncertainty, will result in a flat and wide Beta curve, with an average factor of safety much higher than 1.0 to achieve the same probability of success.

Parametric analyses were performed with the Beta probability distribution for various factors of safety and standard deviations, choosing the minimum and maximum limits for the distribution as the average value $\pm$3 or $\pm$5 standard deviations. Figure 1 shows representative Beta distribution curves for the probability of success of 95% with limits equal to $\pm$3 standard deviations from the average value. Figure 2 shows the variation of the Beta value at the average safety factor for a range of safety factors and success probabilities. Figures 3 and 4, for Beta limits of $\pm$3 and $\pm$5 standard deviations respectively, show a nearly linear relationship between the average factor of safety and the standard deviation for a given success probability. The y-intercept was 1.00 and the coefficient of correlation was greater than 0.998. As the limit of standard deviation approaches zero, the beta curve becomes narrower and steeper, which results in the average factor of safety approaching 1.00.

The engineer may use design charts similar to Figures 3 and 4 (almost identical) to determine the probability of success for the average design safety factor required with a known (or assumed) standard deviation. The stability analysis methods presented by Christian (1997) or Duncan (2000) will help calculate the nominal (average) factor of safety and its standard deviation for given slope conditions. To achieve a greater probability of success, the engineer should alter the design to increase the chosen safety factor, or decrease the variability (through better site characterization, more accurate analyses, ground modification, etc.).

The analyses performed in this section can be applied to other design methods that use a factor of safety approach such as liquefaction evaluation, tieback assessment, and others.

5.1 Slope Stability Example

This example describes a hypothetical slope stability design using electric cone penetration tests, performed during a phase one subsurface investigation, that delineate three geologic strata at the site. The phase two investigation included five borehole shear tests performed in each stratum to estimate the average drained strength parameters and their standard deviations. The borehole shear test accurately measures the drained shear strength of the soil and compares well to laboratory strength tests (Handy, 1986).

The average and standard deviation of the slope's stability were calculated using the point estimate method (Christian, 1997), which performs multiple analyses using permutations of design variables by assigning a value of either the average plus one standard deviation or the average minus one standard deviation to each. Using the shear strength of each of the three strata and the groundwater level as the parametric variables, multiple runs with a Janbu stability analysis program provided a total of 16 permutations (2^n, where n = the number of variables = 4). For these permutations, the average factor of safety equaled 1.25 with a standard deviation of 0.15, and the design chart in Figure 3 indicates an acceptable 95% probability of success.

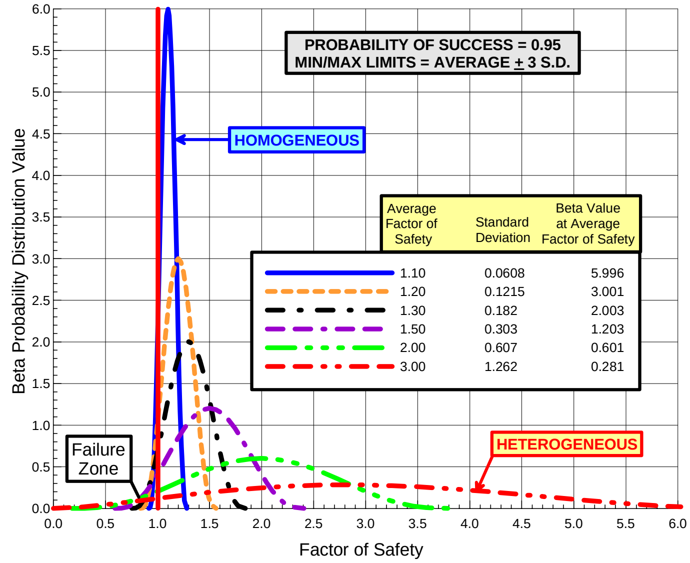

Figure 1: Beta Probability Distribution Curves for Slope Stability

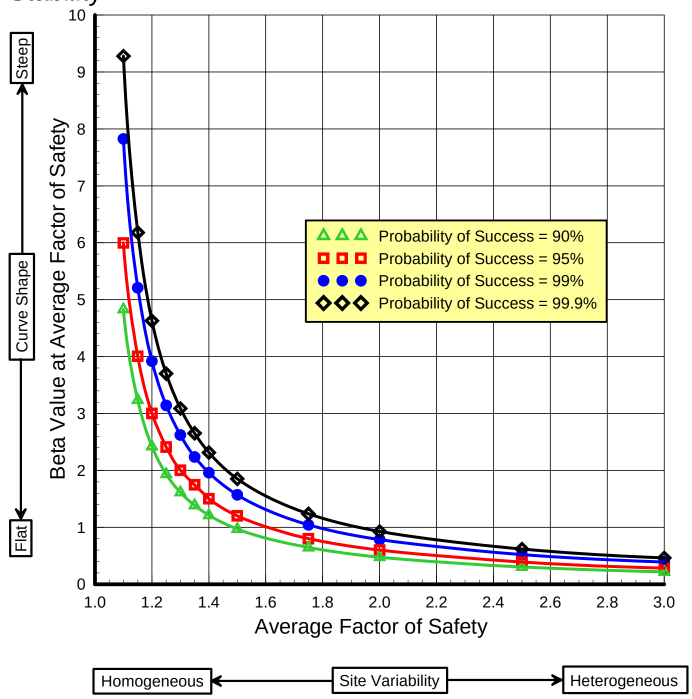

Figure 2: Beta Probability Analyses For Slope Stability with Min/Max Limits = Average ± 3 Standard Deviations

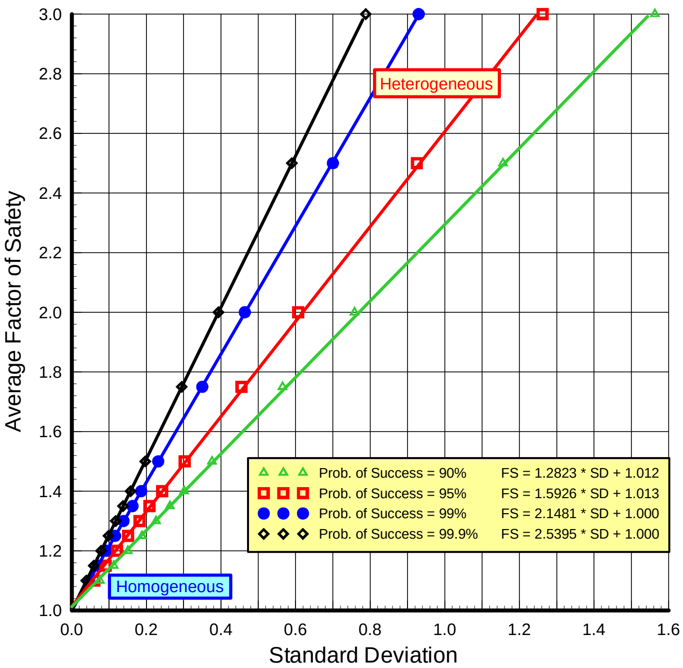

Figure 3: Design Chart of Beta Probability Distribution Analyses for Slope Stability with Min/Max Limits = Average ± 3 Standard Deviations

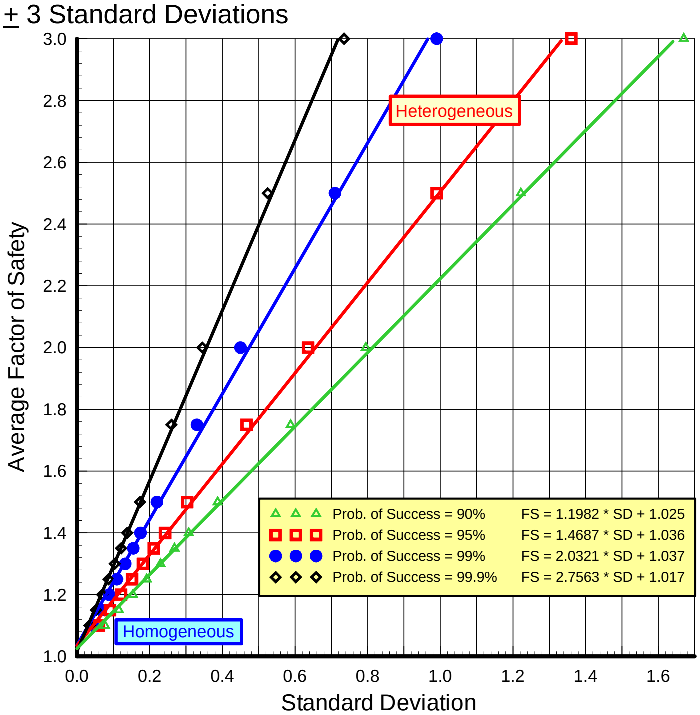

Figure 4: Design Chart of Beta Probability Distribution Analyses for Slope Stability with Min/Max Limits = Average ± 5 Standard Deviations

5.2 *Ground Improvement Evaluation*

In a case study (Miller and Roycroft, 2004) compaction grouting was performed to densify a loose sand to prevent liquefaction of the site. The test program used both 1.2 and 1.5 m spacing between the grouting locations. Afterwards, numerous cone penetration test soundings (CPT) were performed, and the factors of safety against liquefaction were computed. For the 1.5 m spacing the average factor of safety was 1.51, and for the 1.2 m spacing the average factor of safety was 1.65. Based on their engineering judgment, the authors recommended using a minimum acceptable factor of safety of 1.2 and concluded that the 1.5 m spacing was acceptable.

However, from the large amount of data that the authors had collected, the standard deviation of the factor of safety was 0.47 for the 1.5 m spacing and 0.41 for the 1.2 m spacing. These standard deviations values are rather high showing the heterogeneity or high uncertainty of the sands for liquefaction resistance. Presented as Figure 5 are these values plotted on the factor of safety design chart. The risk analyses for 1.5 m spacing yields a probability of success less than 90% (85% numerically computed) and the analyses for 1.2 m spacing yields a probability of success equal to 95%.

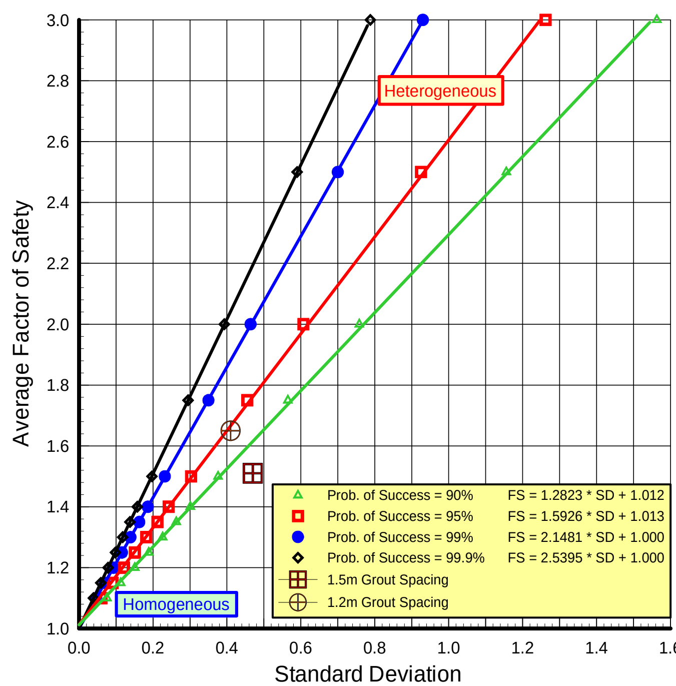

Figure 5: Probability Assessment for Ground Improvement Case History

Beta probability distribution curves can be used to represent both the pile capacity and the load supported by the pile. Where these two curves intersect, the load exceeds the pile capacity and the intersecting area represents the probability of failure. Assigning the average applied load a value of 1.0, and calculating the pile capacity as a multiple of this applied load, leads to a unitless analysis convenient for design purposes. Standard deviations of 0.1 and 0.2 should adequately describe the normal variation of the actual load condition. As found above, minimum and maximum limits of the average value $\pm$ 3 standard deviations should adequately define the expected range of values, with a minimum value of at least zero. Figures 6 and 7 show representative Beta probability distribution curves for a probability of success equal to 95% with load standard deviations of 0.1 and 0.2 respectively.

For homogeneous sites the pile capacity has a low standard deviation, resulting in a narrow, peaked Beta curve with an average value close to the average load. For heterogeneous sites the predicted pile capacity is less accurate and its standard deviation is higher, resulting in a flat and wide Beta curve. For the same level of safety as the homogeneous case, the heterogeneous curve shifts farther to the right of the load curve and has a higher average value.

Figures 8 and 9 show the factor of safety, or pile capacity/load ratio, versus the standard deviation of the pile capacity for standard deviations of the load equal to 0.1 and 0.2, respectively. Again, a nearly linear relationship was found for a given probability of success! For the load standard deviation of 0.2, which contains more uncertainty, the probability of success lines shift up and to the left of the lines for a standard deviation of 0.1. Note that these two figures show a general relationship between the safety factor, the design variability, and the success probability that is valid for any pile design method. These design charts may also be used for tiebacks, soil nails, lateral capacity of piles and other applied load/soil capacity analyses.

6.1 Pile Capacity Example

This example considers twenty (20) cone penetrometer test soundings performed for the hypothetical design of a laboratory foundation supported by steel pipe piles. Column loads will require support of 200 kN per pile, with a standard deviation of 20 kN. For each sounding the LCPC pile capacity prediction method (see Robertson, et al., 1988) provided a pile designed to carry a load of 350 kN (nominal safety factor = 1.75). The design tip elevations for the different column loads in the foundation plan did not vary greatly, resulting in a standard deviation of only 35 kN due to the natural soil variability. Based on a database case study, Robertson, et al. (1988) indicate a coefficient of variation of 0.15 for the LCPC predicted capacity of driven steel pipe piles Using this value, the standard deviation due to the LCPC method is 0.15 * 350 kN = 52.5 kN. The overall standard deviation equals the square root of the sum of the two individual standard deviations squared, or 63.1 kN. The columns were designed to exert a load of 200 kN per pile. Dividing by the 200 kN nominal applied load results in a unitless predicted pile capacity of 1.75, with a standard deviation of 0.32 and load standard deviation of 0.1. Because the building will contain sensitive laboratory equipment, the owner chose a 99% probability of success. However, Figure 8 indicates a probability of success of only 93% for the above parameters.

By increasing the pile diameter so that each pile will have a capacity of 400 kN, the natural standard deviation of 35 kN and the LCPC method standard deviation of 0.15 * 400 kN or 60 kN result in an overall standard deviation of 69.5 kN. Using the unitless values of 2.0 for the factor of safety and 0.35 for pile standard deviation, Figure 8 indicates an acceptable probability of success of 99%.

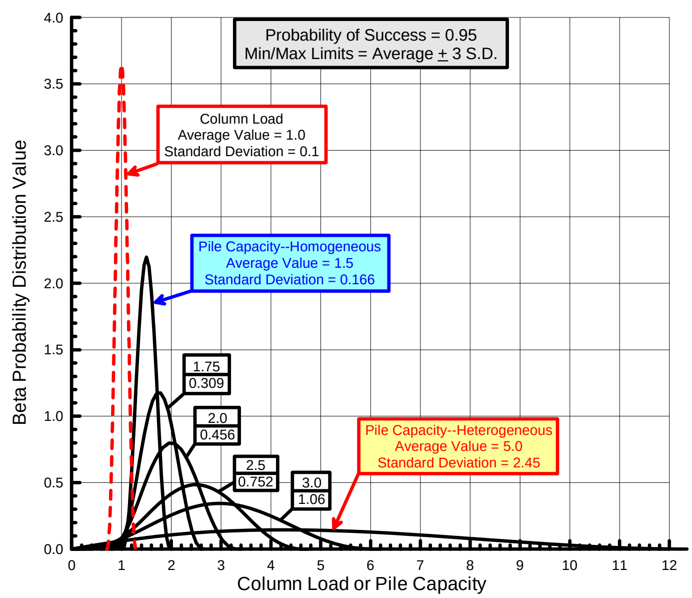

Figure 6: Beta Probability Distribution Curves for Pile Capacity Analyses, Column Load Standard Deviation = 0.1

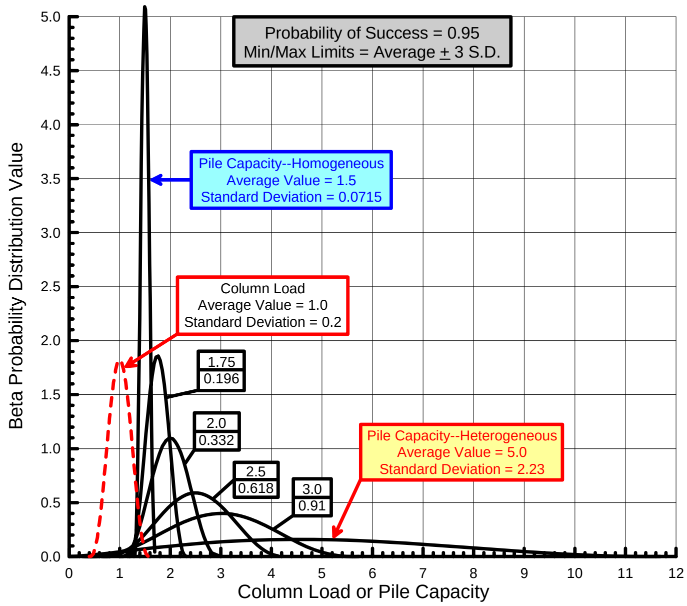

Figure 7: Beta Probability Distribution Curves for Pile Capacity Analyses, Column Load Standard Deviation = 0.2

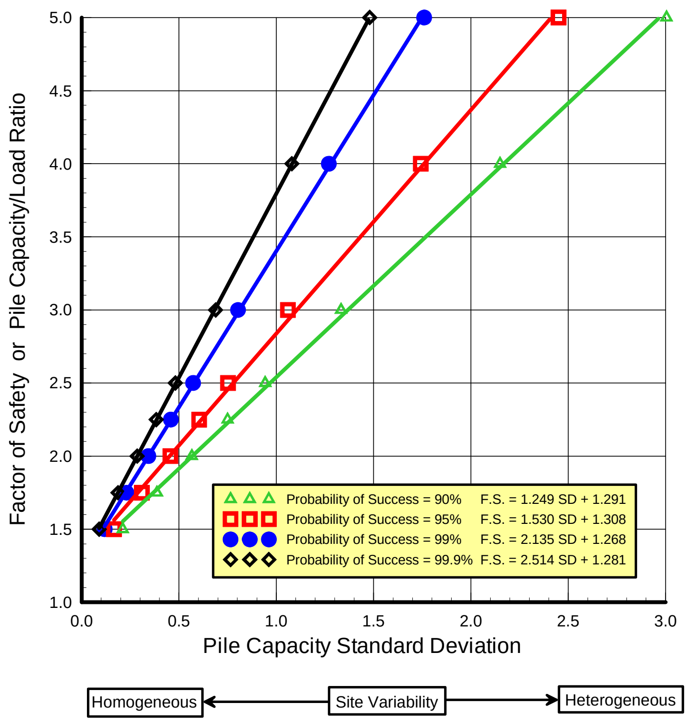

Figure 8: Design Chart for Beta Probability Distribution Analyses of Pile Design, Load Standard Deviation = 0.1

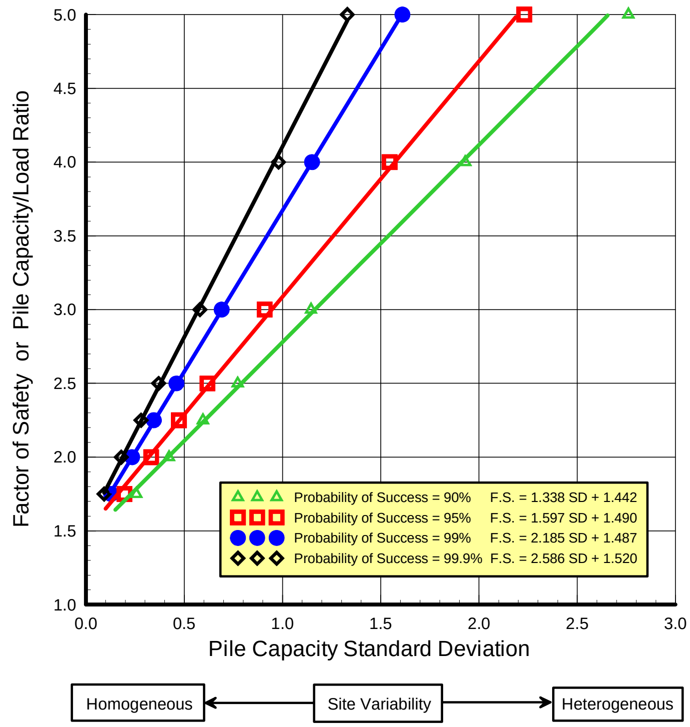

Figure 9: Design Chart for Beta Probability Distribution Analyses of Pile Design, Load Standard Deviation = 0.2

7 SETTLEMENT ANALYSIS

Engineers commonly consider total settlements exceeding 25 mm as unsatisfactory. Design approaches similar to that described below could use a different limit, or could alternatively seek to limit the differential settlement or angular distortion. Using a Beta probability distribution, unacceptable settlement occurs in the zone where it exceeds 25 mm. Of course, the settlement distribution cannot start at less than zero, and again minimum and maximum limits of the average value $\pm$ 3 standard deviations provide reasonable bounds. Figure 10 shows representative Beta distribution curves for a probability of success of 95%, indicating that an increase in the standard deviation requires a decrease in the average settlement to obtain the same success. This requirement may result in a questionable reverse "J" or incorrect "U" shaped Beta distribution due to the high variability. If an unreasonable distribution of this type occurs, the engineer should reduce the variability through better quality testing, increase the allowable settlement threshold to more than 25 mm, or use a different foundation support system.

Figure 11 shows the relationship between the Beta distribution value and the average settlement for several success probabilities of 90, 95, 99 and 99.9%. Figure 12 provides a design chart for settlement analysis. When the beta distribution was "bell-shaped", Figure 12 shows there is a nearly linear relationship between average settlement and its standard deviation for a given probability of success. As the limit of standard deviation approaches zero, the beta curve becomes narrower and steeper, which results in the average settlement (y-intercept) approaching 25 mm.

To determine the standard deviation of settlement for a site, the engineer needs an accurate assessment for static deformation properties of the soils. For most projects requiring settlement assessments, Dilatometer tests (DMT) provide a satisfactory solution. Schmertmann (1986) presents a field-verified settlement calculation method for DMT data. Like traditional settlement predictions based on consolidation tests, this method divides the geologic sections into layers and computes the settlement of these layers. With DMT tests at 20 mm depth intervals, each layer may be as thin as 20-mm, and each DMT sounding provides a separate settlement estimate. The Dilatometer, which is a calibrated static deformation test, accurately predicts settlement and has a coefficient of variation of about 0.18 for all soils except quick silts (Failmezger, Bullock, 2004). By combining all of the settlement predictions, the engineer may compute an average and standard deviation, and then use Figure 12 to determine the probability of not exceeding a threshold limit of 25 mm. If the probability of success is too low, the engineer can perform additional DMT soundings, reduce the applied bearing pressure, or design footings individually.

The Standard Penetration Test (SPT) is also often used for the settlement design of spread footings in sands, particularly in the United States. From case study data, Burland and Burbridge (1985) show that settlement estimates based on the SPT N_{60} value have a coefficient of variation $\approx$ 0.67. This high value probably results from both the inherent variability of the SPT and the use of a dynamic penetration test to estimate static deformation properties (Failmezger, 2001). In Table 1, the upper limits for average settlement are computed for DMT and SPT methods assuming that there is no site variability. At best, the SPT Beta probability distribution has a reverse "J" shape.

	Maximum Average Settlement (mm)	
Probability of Success	DMT Method in all soils with Coeff. of Variation = 0.18	SPT Method in only sands using N_{60} with Coeff. of Variation = 0.67
90%	20.17	12.5
95%	19.28	11.1
99%	18.03	9.5
99.9%	17.15	8.7

Table 1: Maximum Value of Average Settlement with Zero Site Variability for a Threshold Settlement of 25 mm

7.1 Settlement Example

Sixteen dilatometer test soundings were performed for a hypothetical grocery store to depths of approximately 9 m. The soils below about 8 m were dense and no measurable settlement was expected below that depth. Schmertmann's method (1986) provided a settlement estimate for each sounding, with an average settlement of 18 mm and a standard deviation of 4.0 mm due to soil variability. Because the Dilatometer tests were pushed and no quick silts were present at the site, we assume a coefficient of variation for the test and prediction method of 0.18 , resulting in a standard deviation for the DMT prediction method of 0.18 * 18 mm = 3.2 mm. Using an overall standard deviation of 5.1 mm, from Figure 11 the probability of success is slightly more than 90% and is acceptable for the proposed building.

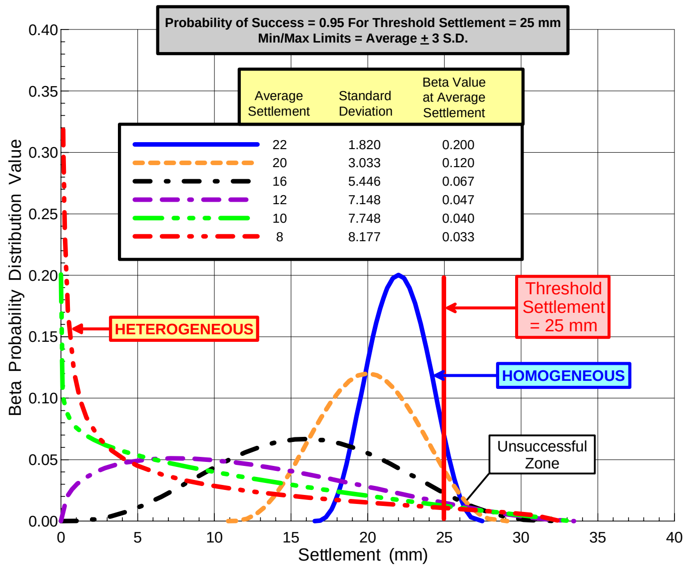

Figure 10: Beta Probability Distribution Curves for Settlement Analyses

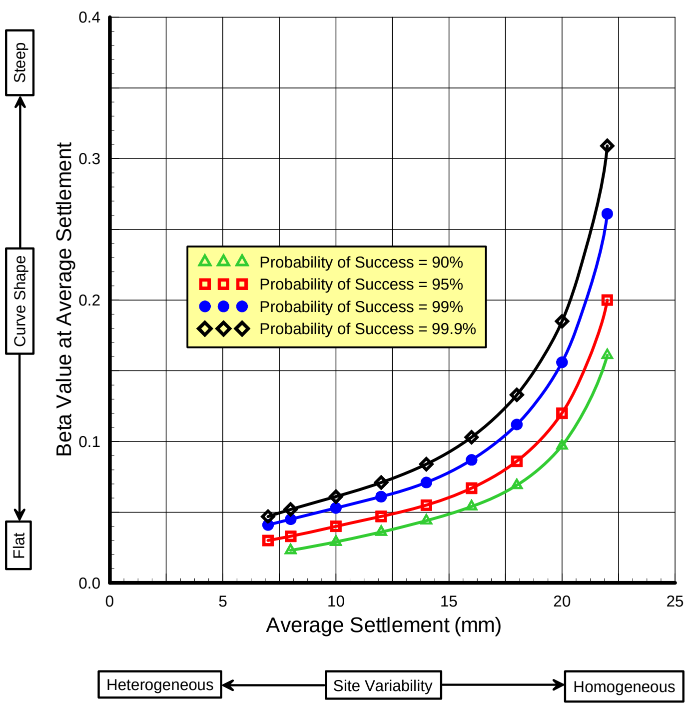

Figure 11: Beta Probability Distribution Analyses for Settlement with an Acceptable Settlement Threshold = 25 mm and Min/Max Limits = Average ± 3 Standard Deviations

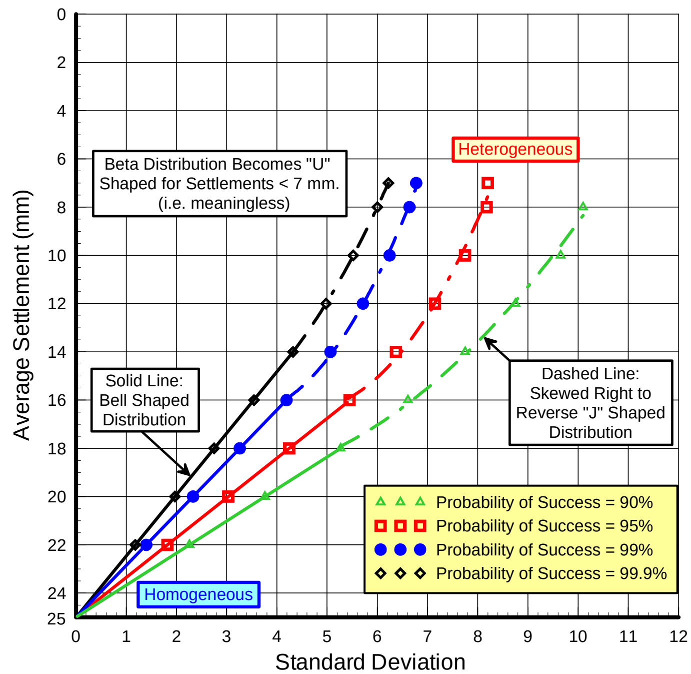

Figure 12: Design Chart for Beta Probability Distribution Analyses for Settlement with a Threshold Settlement = 25 mm and Min/Max Limits = Average ± 3 Standard Deviations

8 CONCLUSIONS/RECOMMENDATIONS

Based on the above examples and our experience with the Beta probability distribution:

1. Good design requires the owner's acceptance and understanding of acceptable risk.
2. Risk can be assessed either qualitatively through engineering judgment or quantitatively through probability. Often engineers have not experienced enough failures to develop good engineering judgment.
3. Effective risk analysis requires the engineer to limit variability, as best possible, to that inherent in the geologic deposit.
4. A thorough and accurate site investigation helps to minimize design variability and improves design efficiency.
5. Soil tests that directly measure design parameters should reduce variability better than empirical correlations with indirect measurements.
6. Design charts based on the Beta distribution provide a simple tool to choose safety factors appropriate to the desired success probability and to the combined level of variation inherent in the design method, site investigation method, and the site itself.
7. For a given probability of success, using the Beta probability distribution within common engineering limits provides a nearly linear relationship between the average value of the design parameter and its standard deviation.

8. The appropriate average factor of safety for slope stability should consider the site variability, consequences of failure and the necessary probability of success.
9. Risk analysis for pile capacity should consider the standard deviation of both the applied load and the soil capacity.
10. Previous database studies show that even in sands settlement analyses based on the Dilatometer test and design methods have much better accuracy than such analyses based on the Standard Penetration Test.

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