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A method for determining the friction angle in sands from the Marchetti dilatometer test (DMT)

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1 INTRODUCTION

The mechanics of the DMT consists of pushing or driving a 94 mm wide, 14 mm thick, steel plate with an approximate 16^{o} cutting edge into the soil and then expanding a 60 mm diameter thin metal membrane, mounted flush on one side of the plate, horizontally against the soil by means of gas pressure. The test operator obtains two pressure readings in approximately 1 minute: the A-pressure required to just begin to move the membrane into the soil and the B-pressure required to move its center 1 mm into the soil. These two pressures provide data relating to the insitu horizontal stress and soil modulus at the test depth. The operator then pushes/drives to the next test depth, usually 0.2 m deeper, and repeats the A- and B-pressure readings, etc., and thus obtains a vertical profile of DMT data -- termed a "DMT sounding".

Marchetti has already written extensively [1,2,3,4] about his flat plate dilatometer test and his semi-empirical methods for predicting important insitu geotechnical parameters such as soil type, lateral stress, preconsolidation stress, undrained strength of clays and drained compressibility modulus. This paper attempts to add a rational method for predicting insitu friction angles for sand to the usefulness of the DMT.

The penetration of the DMT involves a bearing capacity failure of an approximate plane-strain shaped penetrometer (L/B = 7). Such a failure must involve the drained frictional strength ϕ'_{ps} of freely draining coarse soils. Thus, in principle, the bearing capacity of the end area of the dilatometer, denoted q_D, offers the potential for evaluating ϕ'_{ps}.

We know that q_D depends on factors other than ϕ' -- perhaps most importantly on the insitu effective stress conditions during the measurement of ϕ'. The "A" reading from the DMT measures the lateral stress against the sides of the dilatometer after its penetration. The resulting Marchetti K_D factor correlates at least approximately with the insitu horizontal stresses before the penetration. Thus, the DMT provides data about one of the most important, and usually undetermined, variables in determining bearing capacity -- the insitu horizontal stress. The analysis of DMT data therefore offers the possibility of separating the effects of the stress and the ϕ' contribution to bearing capacity. The following describes such a method for determining ϕ'_{ps} from the DMT:

2 USING THE DURGUNOGLU AND MITCHELL (D&M) BEARING CAPACITY THEORY

2.1 Theory accounts for horizontal stress:

These authors developed a theory for the bearing capacity of a rigid wedge in connection with the Apollo lunar exploration program [5,6,7]. This theory improved on the prior ones by Meyerhof and explicitly took account of the important variable of horizontal stress as well as cone shape, point angle, point material, and depth. It seemed to give successful interpretations for relatively shallow depths of penetration (D/B < 30) during its lab verification stage [6], and seemingly reasonable results with the shallow penetrations during the Apollo program. More recent papers, [8,9] have shown that it also appears to give reasonable predictions for the much deeper penetrations involved with static cone and dilatometer testing. Because of this success, and the fact that the theory considers the important horizontal stress variable and the DMT measures this variable, the writer adapted this theory for use with the DMT to predict the plane strain ϕ'_{ps}.

The D&M theory uses equations too complex for hand calculation methods. However, the writer has found then practical for programmable hand calculators for immediate reduction in the field, and for plotters and larger computers in the office.

A number of authors have noted [6,10], that the assumption for the steel/sand friction angle, δ', has an important effect on penetrometer bearing capacity. As recommended in [9], the writer assumed that the steel-soil friction angle $\delta' = 1/2\ \phi'$.

Also, for all DMT penetrations of more than about 0.3 m below the ground surface the bearing capacity of the dilatometer becomes the deep-foundation case and the angle $\beta = \phi'$ in the D&M equations. The lateral stress coefficient, K, needed for the solution by the D&M theory, comes directly from the normal results from the reduction of DMT data. The writer then uses the D&M theory, via an iterative procedure described subsequently, to solve for the plane strain friction angle.

2.2 Theory usually conservative:

In "ordinary" sands the application of the D&M theory may produce too-low ϕ'_{ps} predictions because the actual physical mechanism of wedge penetration for all but the most incompressible sands involves sand compressibility. The wedge penetrates first by forcing volume change before forcing the sand displacement zones assumed in the D&M theory. Thus, and considering only this effect, using the D&M theory with actual bearing capacity data involves using a bearing capacity lower than the theory would predict. This will in turn produce a conservatively too-low ϕ'_{ps} prediction when using that too-low bearing capacity. Reference [10] provides an unusually comprehensive set of q_c vs. depth, friction angle, relative density and horizontal stress data from over 100 carefully controlled tests on one sand in the ENEL calibration test chamber in Milan, Italy. The writer used these data to extract the TABLE 1 comparison between ϕ'_{ax} from triaxial compression tests and from the D&M theory predictions using the test q_c results and the (zero-lateral-strain/constant-vertical-stress) chamber boundary condition. The measured-to-backfigured comparisons in TABLE 1 help to confirm the expected conservatism in ϕ' values predicted from the D&M theory.

The D&M theory does not include the effects of the shear strain, or volume strain, or pore pressure behavior of the cohesionless soil subject to the ϕ' analysis. The D&M theory does not indicate whether to use peak, residual, or in-between ϕ' values. Therefore it does not consider soil structure sensitivity and the possibility of significant progressive action in the bearing capacity failure. Unusually sensitive and/or compressible sands may thus produce D&M theory predicted ϕ'_{ps} values with considerable error. Such soils will produce too-low bearing capacities compared to more usual sands, most likely not because of an actual low ϕ' strength but because unaccounted for structure sensitivity and/or compressibility. A D&M theory-based prediction of ϕ'_{ps} will therefore predict even more conservatively too-low in unusually sensitive and/or compressible sands -- the likely problem sand layers.

In cemented sands and/or strongly dilatent sands the application of the D&M theory, with the tacit assumptions used herein that $c' = 0$ and that penetration induced pore pressures = 0 at the time of the DMT measurements, may result in non-conservative too-high predictions of ϕ'_{ps}. For example: a sand from Tampa Bay, Florida, consisting of quartz and broken shell, gave D&M-DMT predicted ϕ'_{ps} values of 37-54° with an average = 44.4°, from 91 DMTs. This compares with an average CIU ϕ'_{ax} value of 41.5° from 3 tests. Undrained triaxial results from loose, medium, and dense mixed SPT samples of this sand gave ϕ_{ax} = 41.5, 55, and 59.5°, respectively, because of strong dilatency and the resulting negative pore pressures.

TABLE 1 - COMPARISONS BETWEEN MEASURED TRIAXIAL AND D&M THEORY BACKFIGURED VALUES OF ϕ_{ax} FROM CHAMBER TEST DATA 10

Model depth (m)	D_r	σ'_{1o} (bar)	σ'_{ff} assumed = $[1+\sin\phi'_{ps}]$ (bar)	ϕ'_{ax} @ = σ'_{ff} From [10] FIG. 6 (triaxial data)	From [10] FIG. 18 (D&M predicted)	Δ^o (data - predicted)
4	≈ 45%	0.61		41	38 1/2	+2 1/2°
	≈ 70%		1.03	42.3	41 1/2	+0.8°
	≈ 90%	0.65		44 1/2	43 1/2	+1°
7.5		1.14		39.8	38	+1.8°
	do.		1.93	41.1	40 1/2	+0.6°
		1.205		43 1/2	42 1/2	+1°
20		3.0		38	35	+3°
	do.		5.1	39.3	38.3	+1.0°
		3.2		41.9	40.1	+1.8°
33		5.0		37.2	34.0	+3.2°
	do.		8.2	38.4	37.2	+1.2°
		5.3		41.1	38.7	+2.4°
46		7.0		36.5	33.3	+3.2°
	do.		11.4	37.7	36.4	+1.3°
		7.4		40.5	N.A.	

3 ADDITIONAL PENETRATION FORCE MEASUREMENTS AND SEPARATION OF q_D

3.1 Additional thrust measurement:

Because the normal A and B measurements during the DMT do not involve a failure, and the friction angle ϕ' relates to a failure condition, the writer considers it necessary to make an additional measurement with the DMT that involves failure, and suggests simply measuring the total static thrust force required to advance the dilatometer to each new DMT depth. Engineers can determine this value routinely when using the CPT hydraulic equipment to advance the dilatometer. When driving the dilatometer, as with the SPT hammer, the engineer can use the equivalent static force that matches the ENTHRU and blowcount using the methods presented in [11].

Part of the total thrust required overcomes the bearing capacity, q_D, of the approximate 16° dilatometer point over the area of the horizontal projection of the Marchetti dilatometer (1280 mm²). Assuming the existance of this total thrust data, the evaluation of q_D then becomes a problem of extracting this bearing capacity from the total thrust.

FIGURES 1 and 2 show examples of comparative sets of logs, about 1 m. apart - one of q_c and the other the q_{Dp} required to push down the dilatometer and its pushrods (3.6 mm diameter as ordinarily used with the CPT). The parallelism between q_c and q_{Dp}, despite the q_c fluctuations, suggests only small pushrod friction effects and that a properly corrected q_{Dp} might provide acceptable estimates of q_D.

3.2 The net bearing capacity force:

FIGURE 3 herein illustrates the penetrating dilatometer with the forces that act on it in a vertical direction. The sum of these forces must equal 0. Because the DMT A-pressure reading gives the horizontal soil pressure against the dilatometer blade, the engineer can make a good estimate of all the forces involved except one -- F_r, the total friction force between the soil and the rods above the friction reducing ring section above the dilatometer. A subsequent section will show that the engineer can often reasonably assume this force as $F_r = 0$. With this assumption he or she can conveniently sum the forces and extract the end bearing force F_e, which when divided by the dilatometer end area of 1280 mm² gives q_D.

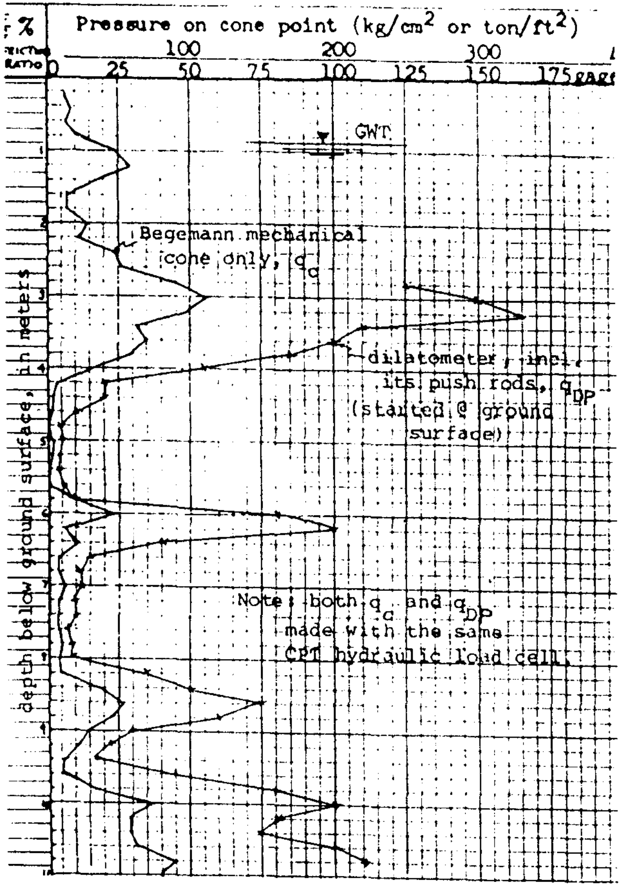

FIGURE 1 - Example from a cooling tower site in Jacksonville, FL

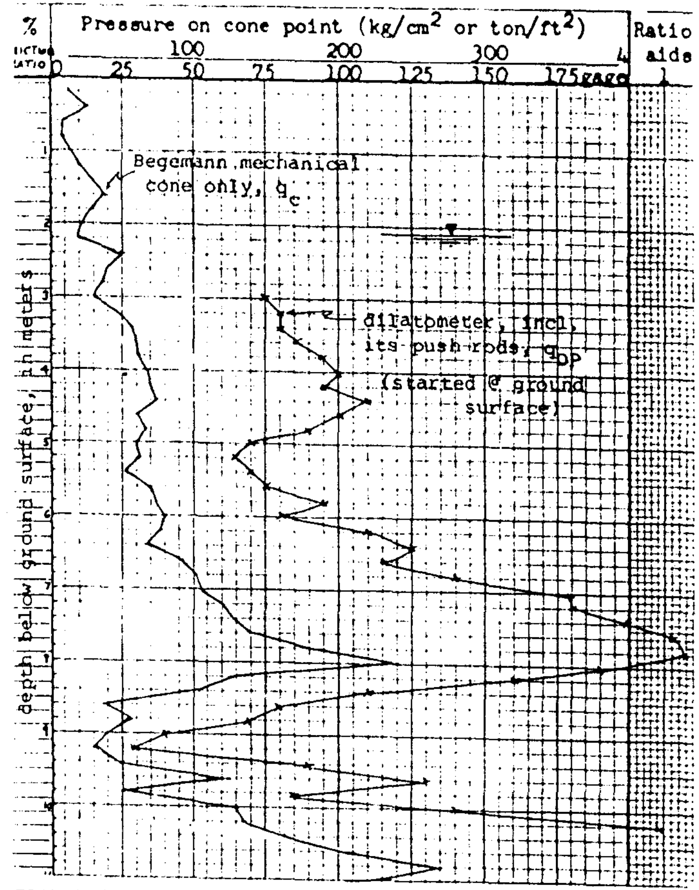

FIGURE 2 - Example from another cooling tower site in Jacksonville, FL

COMPARISONS OF TOTAL THRUST TO ADVANCE DILATOMETER & CPT CONE

4 COMPARING THE CPT q_c AND THE DMT q_D

4.1 Theorectically:

The plane strain D&M theory also permits the calculation of q_c by means of a D&M shape factor correction and thus allows the engineer to compare q_D and q_c over any desired range of the key variables involved -- namely, ϕ', D/B, and K. D&M used the same equation (1)

$$q = \gamma B N_{\gamma q} \xi_{\gamma q} \quad (1)$$

for wedge and cone penetrometers in cohesionless soils, with their different bearing capacities accounted for by the $\xi_{\gamma q}$ shape factor. The shape factor thus includes the effects of any differences between ϕ'_{ps} and ϕ_{ax} that might apply between wedge and cone behavior.

The writer used their theory to solve for $N_{\gamma q}$ and $\xi_{\gamma q}$ and obtained the results shown in TABLE 2. The reader will see that the ratio q_D/q_c does not vary significantly with K_o and D/B. For the purposes herein it seems reasonable to assume that the ratio varies with only ϕ'. FIGURE 4 shows this variation and suggests a simple linear equation (2) to express the variation.

TABLE 2 - PARAMETRIC STUDY, USING THE D&M THEORY FOR THE q_D/q_c RATIO

ϕ'	K_o	D/B	pt. angle	$N_{\gamma q}$	Ratio	$\xi_{\gamma q}$	ave(q_D/q_c)
25°	0.577	50	16° (DMT) 60° (CPT)	598 454	1.32	1.010	
		800	16 60	9,405 7,220	1.30		1.29
	5	50	16 60	1,077 833	1.29		
		800	16 60	17,395 13,370	1.30		
30°	0.5	50	16 60	1,080 845	1.28	1.237	
		200	16 60	4,300 3,370	1.28		1.03
	2	50	16 60	1,510 1,200	1.26		
		200	16 60	6,100 4,800	1.27		
40°	0.5	50	16 60	4,370 3,910	1.12	1.549	
		200	16 60	18,220 15,900	1.15		
	2	50	16 60	7,350 6,815	1.08		0.73
		200	16 60	31,985 28,125	1.15		
45°	0.707	50	16 60	10,960 11,370	0.96	2.200	
		800	16 60	204,960 193,285	1.04		0.45
	2	50	16 60	17,915 19,445	0.92		
		800	16 60	358,400 338,600	1.06		

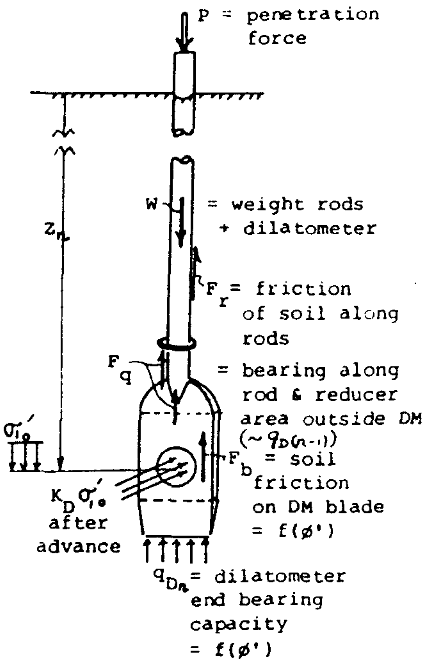

FIGURE 3 - VERTICAL FORCES ON THE ADVANCING DILATOMETER

The results in TABLE 2 show that the q_D/q_c ratio falls on both sides of 1.0, depending on the value of ø'. If one accepts the validity of this theory for the q_D/q_c ratio (the same theory used to extract $ø'_{ps}$ from the DMT) it becomes possible to solve for the F_r force for those cases where we have parallel CPT and DMT data. In this way the writer determined, as detailed subsequently, that the value of F_r in primarily sand soils probably has so small a value that one may adequately, but admittedly non-conservatively, simplify and assume $F_r = 0$.

4.2 Field measurements of q_D/q_c:

TABLE 3 presents a summary of the comparisons obtained between the computer q_D (theory)/q_c(measured) ratios for various assumed F_r and those predicted from the above theory when using the $ø'_{ps}$ value obtained by applying the theory to the DMT and total thrust data at one sandy soil site in Florida. The writer expressed the F_r force as a fraction, R, of rod friction as expected from the CPT friction ratio data. These comparisons cover a considerable range of depth and soil bearing strength in both silty and clean sands, but from only one site in Florida. The writer then added the TABLE 3 comparisons to FIGURE 4.

TABLE 3 - CALCULATIONS TO DETERMINE THE LIKELY BEST R-FRACTION OF CPT PUSHROD FRICTION EFFECTIVE ABOVE THE DMT FRICTION REDUCER (using R_f = 1.0%)

Depth (m)	R	D&M $ø'_{ps}$	DMT q_D (b)	CPT** q_c (b)	(q_D/q_c) DMT	(q_D/q_c) D&M theory
3.6	✓0	33.9	54	68	0.79	✓ 0.94
	0.1	33.4	47	"	0.69	0.96
	0.2	32.7	41	"	0.60	0.99
7.8	✓0	38.9	137	185	0.74	✓ 0.74
	0.1	38.2	112	"	0.61	0.77
	0.2	36.9	89	"	0.49	0.82
9.2	✓0	29.0	18.4	39	0.47	✓ 1.14
	0.1			"		
10.2	0	40.1	140	130	1.08	0.70
	✓0.1	38.5	107	"	0.82	✓ 0.76
	0.2	36.1	70	"	0.54	0.86
4.6	0✓	31.7	36	54	0.67	✓ 1.03
6.6	0✓	21.6	7.7	17	0.45	✓ 1.44
10.6	0✓	36.5	81	87	0.93	✓ 0.84
	0.1	35.8	64	"	0.74	0.87

✓ denotes best comparison

$$F_r = \pi d R \int_1^2 R_f\, q_c\, \Delta Z \quad \text{. . . . . . eqn. (3)}$$

1 = frict. reducer above DM blade
2 = ground surface

where d = pushrod diameter
R_f = CPT local friction ratio
R = assumed portion of CPT local friction effective above reducer

** Measured using Begemann mechanical tip

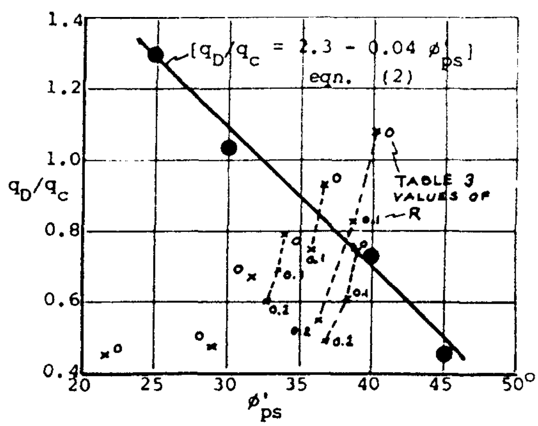

FIGURE 4 - PREDICTED q_D/q_c FROM D&M THEORY & COMPARED WITH TABLE 3 VALUES

The final 3 columns in TABLE 3 and the graphical comparison on FIGURE 4 show that for these comparisons the use of $R = 0$, meaning $F_r = 0$, produces the best match between the DMT-computed and D&M-computed values of the (q_D/q_c) ratio. Because of this check, and many seemingly reasonable predictions of ϕ'_{ps} at other sites when using $R = 0$, the writer tentatively recommends using the simplification that $R = 0$, or $F_r = 0$. However, considering only this assumption, and the fact that some positive F_r must exist, the assumption produces an error in ϕ'_{ps} in the non-conservative direction. This will tend to reduce the conservatism discussed in Sections 2.2 and 5.

5 THE ITERATIVE CALCULATION PROCEDURE

From eqn. (1) the value of q_D results from the direct application of the D&M theory after solving for the intermediate parameter $N_{\gamma q}$. The geometry of the dilatometer gives the angles involved in the D&M equations for $N_{\gamma q}$, with the exception of ϕ'_{ps}. On the other hand, the described method for evaluating q_D from the total thrust data also involved determining the plate friction force, F_p (see FIG. 3). This force also depends on $\delta' = \phi'_{ps}/2$. Thus, the present method produces two evaluations of q_D, which one then equates. This produces only one unknown ϕ'_{ps}, on both sides of an equation. By using trial values of ϕ'_{ps} in suitable increments (the writer uses 1^o), and a linear interpolating over that increment for when the value of the calculated ϕ'_{ps} = the assumed ϕ'_{ps}, one obtains the value of ϕ'_{ps}. This value of ϕ'_{ps} produces added conservatism because of a linear interpolation (up to 0.2^o for a 1^o increment).

6 EXAMPLES OF LOGS OF ϕ'_{ps} DETERMINED USING THE PROPOSED METHOD

In addition to the computed ϕ'_{ax} and ϕ'_{ps} results presented in TABLES 1 and 3, the writer has included other examples of the results of the proposed method in the form of depth-logs of ϕ'_{ps}.

FIGURE 5 shows an example from Jacksonville, Florida, together with the q_c log from a parallel CPT sounding. Note how the predicted ϕ'_{ps} pattern follows the q_c pattern, and the seemingly reasonable 27-40^o range of the ϕ'_{ps} results.

FIGURE 6 shows DMT results from New Mexico, USA, at a test site to check the compaction of a fine sand layer by dynamic compaction (DC). (a) correctly shows no change in the material index, (b) shows an increase in K_D, and therefore horizontal stress. (c) shows an average 4^o increase in ϕ'_{ps}. (d) shows a large decrease in compressibility. As illustrated by this example, the DMT and the proposed method for determining ϕ' apparently permit the separation of the stress and strength and compressibility changes that result from a soil treatment such as dynamic compaction.

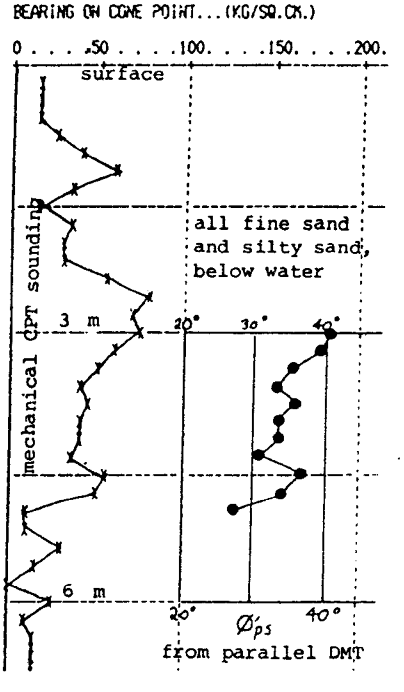

FIGURE 5 - EXAMPLE FROM JACKSONVILLE, FLORIDA, SHOWING COMPARSION BETWEEN MEASURED q_c SOUNDING LOG AND CALCULATED ϕ'_{ps} USING THE PROPOSED METHOD

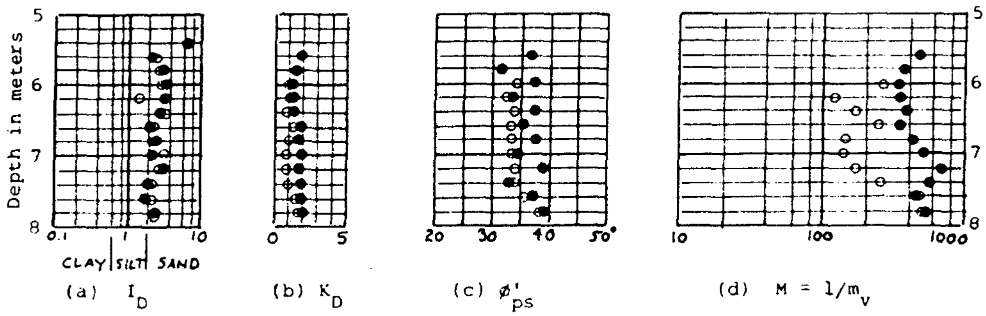

FIGURE 6 - EXAMPLE OF DMT RESULTS BEFORE (○) AND AFTER (●) DYNAMIC COMPACTION FROM A SITE IN NEW MEXICO, SHOWING COMBINED EFFECTS OF STRESS AND STRENGTH INCREASES

7 ESTIMATED ϕ'_{ax} VS. ϕ'_{ps} VS. STRESS LEVEL

The proposed method of calculations ϕ' from the DMT results obtains essentially the plane strain ϕ'_{ps} because the D&M theory analyses the plane strain case and the width/thickness ratio of the dilatometer blade = 7.

Sometimes engineers want ϕ'_{ps} values, as for conventional 2D slope stability analyses or for the bearing capacity of long footings. Sometimes they want the axisymmetric ϕ'_{ax} values, as for the bearing capacity of circular or square footings. Laboratory triaxial tests measure ϕ'_{ax}. $\phi'_{ps} \geq \phi'_{ax}$ because of the plane strain constraint in the 3rd dimension. References [12,13], among others, discuss the relationship between the two angles. The writer suggests the following eqn. (3) for an approximate comparison between peak ϕ' values:

$$\phi'_{ax} \approx \phi'_{ps} - \frac{D_r\%}{100}\,(10^\circ) \quad \ldots\ldots\ldots (4)$$

Engineers now commonly recognize the Mohr-Coulomb failure envelope for soils, including sands, has a distinct curvature and that when $c' = 0$ the secant ϕ' values will decrease with increasing stress. Each ϕ'_{ps} result from DMT data as determined by the proposed method assumes $c' = 0$ and represents a secant value associated with a particular "average" normal stress level on the D&M theory failure planes around the penetrating wedge. Durgunoglu and Mitchell did not determine such an average stress. The writer has assumed this stress on the "average" failure plane as the Rankine passive stress, which = (vertical overburden effective stress) x (1 + sin ϕ'_{ps}). The writer assumed this for the TABLE 1 comparisons and they produced the reasonable, conservative predictions shown.

8 CONCLUSIONS

The writer has used a bearing capacity theory made and verified by others to evaulate ϕ'_{ps} in coarse to silty sands. The application of this theory allows the prediction of the plane strain ϕ'_{ps} value from a Marchetti dilatometer test when the dilatometer penetrates cohesionless soils with the penetration force measured or estimated. The method involves the application of complex formulas and therefore requires programming into modern hand-held calculators or larger computers.

The writer believes the proposed DMT-ϕ' method capable of insitu evaluations of peak ϕ'_{ps} and ϕ'_{ax} with acceptable, usually conservative, accuracy for many engineering purposes. But, he has used the method for less than two years, and only in the USA and Canada. It has produced apparently good, but usually unchecked results. The reader should consider it suitable for trial applications.

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10 NOTATION USED:

B = least width or diameter of penetrometer
c' = Mohr-Coulomb effective cohesion intercept
D = depth of embedment of bearing capacity surface, from ground surface or diameter
D_r = relative density, void ratio basis
K, K_o = lateral effective stress coefficient
$N_{\gamma q}$ = bearing capacity factor used in D&M theory
q_c = static cone penetration (CPT) bearing capacity
q_D = dilatometer blade (DMT) bearing capacity
Z = depth below ground surface
β = angle in D&M theory associated with depth of embedment
γ = soil unit weight
δ' = effective friction angle between sand and penetrometer surface
$\xi_{\gamma q}$ = cone shape factor used in D&M Theory
σ'_{ff} = average normal effective stress on the failure plane generated by the advancing DMT blade producing a continuous bearing capacity failure.
δ'_{1o} = initial vertical effective stress at depth = D
ϕ' = effective soil friction angle
ϕ'_{ax} = effective soil friction angle, axisymmetric case
ϕ'_{ps} = effective soil friction angle, plane strain case
α = half-angle of penetrometer cutting edge

ABBREVIATIONS:

CIU = consolidated istropically, undrained compression triaxial test
CPT = static cone penetration test
DC = dynamic compaction or dynamic consolidation
D&M = Durgunoglu and Mitchell
DMT = Marchetti flat plate dilatometer test
ENTHRU = SPT hammer energy passing thru delivery system and reaching sampler